BWM: Best Worst Method

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MCDM problem

- **Decision-making** is generally defined as the cognitive process of selecting an alternative from a set of alternatives.
- A multi-criteria decision-making (MCDM) problem is a problem where a decision-maker has to find the best alternative from a set of alternatives considering a set of criteria.



MCDM problem (example 1)

A shipper has to select the best port among a set of four ports: Le Havre (France); Antwerp (Belgium); Rotterdam (Netherlands); Hamburg (Germany)



Considering the criteria:

- port efficiency
- port infrastructure
- location
- port charges
- interconnectivity

MCDM problem (example 2)



A manufacturer has to select the best location for its central warehouse from a set of three alternative locations:



Utrecht; Arnhem; Dordrecht

Considering the criteria:

- proximity to customers
- proximity to suppliers
- investment costs
- expansion possibility
- road connection
- rail connection

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water connection

MCDM problem (example 3)

I want to buy a car from the following set:



Considering the criteria: • price

- quality
- comfort
- safety
- style



MCDM problem (formulation)

A discrete MCDM problem is generally shown as a matrix as follows:

$$P = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ a_1 \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_m \begin{pmatrix} p_{m1} & p_{m2} & \cdots & p_{mn} \end{pmatrix}$$

where

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 $\{a_1, a_2, \dots, a_m\}$ is a set of feasible alternatives (actions, stimuli), $\{c_1, c_2, \dots, c_n\}$ is a set of decision-making criteria, and p_{ii} is the score of alternative *i* with respect to criterion *i*

 p_{ij} is the score of alternative *i* with respect to criterion *j*.

MCDM problem (goal)

The **goal** is to select^{*} the best (e.g. most desirable, most important) alternative (an alternative with the best overall value V_i)

(*other goals: rank, sort)

$$V_i = \sum_{j=1}^n w_j p_{ij}$$
$$w_j \ge 0, \sum w_j = 1$$

The scores P_{ij} are collected from available data sources (if they are objective and available, e.g. price), or measured using qualitative approaches (e.g. Likert scale), or calculated similar to the weights of the criteria (if they are subjective, e.g. quality) and normalized using a normalization formula.



BWM (pairwise comparison)

BWM uses pairwise comparison* to find the weights (w_i) of the criteria.

- Pairwise comparison a_{ij} shows how much the decision-maker prefers criterion *i* over criterion *j*.
- To show such preference, we may use Likert scales (e.g. very low ... very high) with a corresponding numerical scale like:
- 0.1, 0.2, ..., 1 (0.1: equally important, ..., 1: *i* is extremely more important than *j*).
- 1, 2, ..., 100 (1: equally important, ..., 100: *i* is extremely more important than *j*).
- 1, ...,9 (1: equally important, ..., 9: *i* is extremely more important than *j*).

* Thurstone, L.L. (1927). A law of comparative judgement. Psychological Review, 34, 278–286.

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• Determine a set of decision criteria.

In this step, the decision-maker considers the criteria $\{c_1, c_2, \dots, c_n\}$ that should be used to arrive at a decision. For instance, in the case of buying a car, the decision criteria can be:

{quality (c_1) , price (c_2) , comfort (c_3) , safety (c_4) , style (c_5) }

It is clear that for different decision-makers, the set of decision criteria might vary.



- Determine the best (e.g. the most important), and the worst (e.g. the least important) criteria.
- In this step, the decision-maker identifies the best and the worst criteria. No comparison is made at this stage. For example, for a particular decision-maker, $price(c_2)$ and $style(c_5)$ may be the *best* and the *worst* criteria respectively.



 Determine the preference of the best criterion over all the other criteria using a number between 1 and 9 (or other scales).
The resulting Best-to-Others (BO) vector would be:

$$A_{B} = \left(a_{B1}, a_{B2}, \dots, a_{Bn}\right)$$

where a_{Bj} indicates the preference of the best criterion *B* over criterion *j*. For our example, the vector shows the preference of price(c_2) over all the other criteria.



 Determine the preference of all the criteria over the worst criterion using a number between 1 and 9 (or other scales).
The resulting Others-to-Worst (OW) vector would be:

$$A_{W} = (a_{1W}, a_{2W}, \dots, a_{nW})^{T}$$

where a_{jW} indicates the preference of the criterion j over the worst criterion W. For our example, the vector shows the preference of all the criteria over style(c_5).



• Find the optimal weights $(w_1^*, w_2^*, \dots, w_n^*)$

The optimal weights for the criteria is the one where, for each pair of w_B/w_j and w_j/w_W , we have $w_B/w_j = a_{Bj}$ and $w_j/w_W = a_{jW}$.

To satisfy these conditions for all *j*, we should find a solution where the maximum absolute differences $\left|\frac{w_B}{w_j} - a_{Bj}\right|$ and $\left|\frac{w_j}{w_W} - a_{jW}\right|$ for all *j* is minimized.



Steps of BWM: Step 5, *min-max*

To find the optimal weights, the following optimization model is formulated.

$$\min_{j} \max_{j} \left\{ \left| \frac{w_{B}}{w_{j}} - a_{Bj} \right|, \left| \frac{w_{j}}{w_{W}} - a_{jW} \right| \right\}$$

s.t.
$$\sum_{j} w_{j} = 1$$

 $w_{j} \ge 0$, for all j (1)



Steps of BWM: Step 5, Converted *min-max*

Model (1) is converted to the following model.

 $\min \xi$ s.t. $\left|\frac{w_B}{w_i} - a_{Bj}\right| \le \xi$, for all j $\left|\frac{w_j}{w_W} - a_{jW}\right| \le \xi, \text{ for all } j$ $\sum_{i} w_{i} = 1$ $w_i \ge 0$, for all j

(2)

Solving Model(2), the optimal weights $(w_1^*, w_2^*, ..., w_n^*)$ are obtained.



Consistency ratio: Definition

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• **Definition**. A comparison is fully consistent when $a_{Bj} \times a_{jW} = a_{BW}$ for all *j*, where a_{Bj} , a_{jW} , a_{BW} are respectively the preference of the best criterion over the criterion *j*, the preference of criterion *j* over the worst criterion, and the preference of the best criterion over the worst criterion.



Tallest (B) j Shortest (W)

Consistency ratio: A robust index

As it is likely that we do not have the full consistency, we can calculate the level of consistency using a robust index called consistency ratio:

a_{BW}	2	3	4	5	6	7	8	9
Consistency Index	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23
$(\max \xi)$								

 $Consistency Ratio = \frac{\xi^*}{Consistency Index}$

Consistency ratio (CR) \in [0, 1]. The lower the CR the more consistent the comparisons, hence the more reliable results.



Consistency ratio: Thresholds

Criteria <i>a_{BW}</i>	3	4	5	6	7	8	9
3	0.2087	0.2087	0.2087	0.2087	0.2087	0.2087	0.2087
4	0.1581	0.2352	0.2738	0.2928	0.3102	0.3154	0.3273
5	0.2111	0.2848	0.3019	0.3309	0.3479	0.3611	0.3741
6	0.2164	0.2922	0.3565	0.3924	0.4061	0.4168	0.4225
7	0.2090	0.3313	0.3734	0.3931	0.4035	0.4108	0.4298
8	0.2267	0.3409	0.4029	0.4230	0.4379	0.4543	0.4599
9	0.2122	0.3653	0.4055	0.4225	0.4445	0.4587	0.4747

For instance, if we have a problem with 6 criteria and the maximum value in the pairwise comparison system is 7, then the threshold is 0,3931, which implies that values of CR below 0,3931 are acceptable for such a problem.



BWM: post-optimality

When we have more than 3 criteria AND a consistency ratio grater than zero, Model (2) has multiple optimal solutions. Solving the following two models the lower bound and the upper bound of the weights are obtained:



(3b)

A decision-maker selects an optimal solution from the interval weights. This can be the center of the intervals, for instance.

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Linear BWM, *min-max*

We can also minimize the maximum from the set $\{|w_B - a_{Bj}w_j|, |w_j - a_{jW}w_W|\}$ which results in the following model:

$$\min \max_{j} \left\{ |w_{B} - a_{Bj}w_{j}|, |w_{j} - a_{jW}w_{W}| \right\}$$

s.t.
$$\sum_{j} w_{j} = 1$$

 $w_{j} \ge 0$, for all j (4)



Linear BWM, Converted *min-max*

Model (4) is converted to the following model.

 $\min \xi^{L}$ s.t. $\left|w_{B} - a_{Bj}w_{j}\right| \leq \xi^{L}, \text{ for all } j$ $\left|w_{j} - a_{jW}w_{W}\right| \leq \xi^{L}, \text{ for all } j$ $\sum_{j} w_{j} = 1$ $w_{j} \geq 0, \text{ for all } j$ (5)

This is a linear model with a unique solution $(w_1^*, w_2^*, ..., w_n^*)$.

 ξ^{L^*} is considered as a good indicator of the consistency* of the comparisons.

(*note: the ξ^{L^*} of Model (5) should not be divided by the consistency index values on page 17)

Model (5) is a good linear *approximation* of Model(2).



An example (buying a car)

For buying a car, a buyer considers five criteria: quality (c_1) , price (c_2) , comfort (c_3) , safety (c_4) , and style (c_5) . The buyer provides the following pairwise comparison vectors (BO: Best to Others; OW: Others to Worst).

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BO	\rightarrow	Quality	Price	Comfort	Safety	Style
Best criterion: Pric	e	2	1	4	3	8

OW	Worst criterion: Style
Quality	4
Price	8
Comfort	4
Safety	2
Style	1

An example (buying a car, Model (2))





An example (buying a car: Model (2))

1. Suppose we have a performance matrix as follows (10-pont scale; the higher the better):

 $\begin{array}{ccccc} ql & pr \ cm \ sf \ st \\ N \begin{pmatrix} 8 & 6 & 5 & 8 & 8 \\ 7 & 9 & 6 & 7 & 7 \\ H \begin{pmatrix} 8 & 4 & 7 & 6 & 6 \end{pmatrix} \end{array}$

2. And here are the weights we get from page 22 (center of the intervals):

 $w^* = \{0.2024, 0.4609, 0.1536, 0.1345, 0.0512\}$

3. Then we can get the overall value of each car using the following function :

 $V_i = \sum_{j=1}^n w_j p_{ij}$

If the alternative scores (performance matrix) are of from different scales (ton, euro, km) we first normalize the scores: $x_k^{norm} = \begin{cases} \frac{x_k}{\max\{x_i\}}, & \text{if x is positive (such as quality),} \\ 1 - \frac{x_k}{\max\{x_i\}}, & \text{if x is negative (such as price).} \end{cases}$

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 $V_N = (8 \times 0.2024) + (6 \times 0.4609) + (5 \times 0.1536) + (8 \times 0.1345) + (8 \times 0.0512) = 6.6382$

This car, with the highest overall score, is selected.



 $V_{R} = (7 \times 0.2024) + (9 \times 0.4609) + (6 \times 0.1536) + (7 \times 0.1345) + (7 \times 0.0512) = 7.7864$



 $V_{H} = (8 \times 0.2024) + (4 \times 0.4609) + (7 \times 0.1536) + (6 \times 0.1345) + (6 \times 0.0512) = 5.6522$



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An example (buying a car: Model (5))



If we solve the problem using the Linear BWM (Model (5)) we get the following weights:

 $w^* = \{0.246, 0.431, 0.154, 0.123, 0.046\}$

With the following consistency ratio:

 $\xi^{*L} = 0.061$

As can be seen the weights are slightly different from the center of intervals, yet we come to the same best decision. $V_{N} = (8 \times 0.246) + (6 \times 0.431) + (5 \times 0.154) + (8 \times 0.123) + (8 \times 0.046) = 6.676$

Again, this car, with the highest overall score, is selected.



 $V_R = (7 \times 0.246) + (9 \times 0.431) + (6 \times 0.154) + (7 \times 0.123) + (7 \times 0.046) = 7.708$



 $V_{H} = (8 \times 0.246) + (4 \times 0.431) + (7 \times 0.154) + (6 \times 0.123) + (6 \times 0.046) = 5.784$



Bayesian BWM (A group decisionmaking model)



$$P\left(w^{agg}, w^{1:K} \mid A_B^{1:K}, A_W^{1:K}\right) \propto P\left(A_B^{1:K}, A_W^{1:K} \mid w^{agg}, w^{1:K}\right) P\left(w^{agg}, w^{1:K}\right)$$
$$= P(w^{agg}) \prod_{k=1}^K P\left(A_W^k \mid w^k\right) P\left(A_B^k \mid w^k\right) P\left(w^k \mid w^{agg}\right).$$

$$\begin{array}{c} 1:K\\W \end{array} \right) \qquad \qquad A_B^k \mid w^k \sim multi \\ A_W^k \mid w^k \sim mult \end{array}$$

$$P(x) = \sum_{y} P(x, y)$$
$$P\left(A_{W}^{k} \mid w^{agg}, w^{k}\right) = P\left(A_{W}^{k} \mid w^{k}\right)$$

$$A_B^k \mid w^k \sim multinomial(1/w^k)$$
$$A_W^k \mid w^k \sim multinomial(w^k)$$

$$w^k \mid w^{agg} \sim Dir(\gamma \times w^{agg})$$

$$\gamma \sim gamma(a, b)$$

$$w^{agg} \sim Dir(\alpha)$$



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Bayesian BWM (A group decisionmaking model): Credal Ranking

$$P(c_i > c_j) = \int I_{(w_i^{agg} > w_j^{agg})} P(w^{agg})$$

$$\begin{split} P(c_i > c_j) &= \frac{1}{Q} \sum_{q=1}^Q I(w_i^{agg_q} > w_j^{agg_q}) \\ P(c_j > c_i) &= \frac{1}{Q} \sum_{q=1}^Q I(w_j^{agg_q} > w_i^{agg_q}) \end{split}$$



Bayesian BWM (A group decisionmaking model): Example

Sample size = 50



Salient features: Data efficiency





Salient features: Structured pairwise comparison

• BWM makes the comparisons in a structured way, which makes the judgment easier and more understandable, and more importantly leads to more consistent comparisons, hence more reliable weights/rankings.

In our daily life we usually compare things to some reference points!







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Salient features: Debiasing Strategy

• It has the debiasing strategy "consider-the-opposite"









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Highlights

- BWM is an easy-to-understand and easy-to-apply method.
- BWM makes the comparisons in a structured way.
- BWM leads to more consistent comparisons, hence more reliable weights/rankings.
- BWM is suitable for both situations: flexibility is desirable (nonlinear BWM); flexibility is not desirable (linear BWM).
- BWM can supports a single decision-maker.
- BWM is also proper for group decision-making.
- BWM supports reaching consensus in a natural way.
- BWM is efficient in terms of input data.
- BWM can be applied to different MCDM problems with qualitative and quantitative criteria.
- BWM is compatible with many other existing MCDM methods.



Want to know more!?

For more information you may read these papers:

- Rezaei, J. (2015). Best-worst multi-criteria decision-making method, *Omega*, Vol. 53, pp. 49–57.
- Rezaei, J. (2016). Best-worst multi-criteria decision-making method: Some properties and a linear model, *Omega*, Vol. 64, pp. 126-130.
- Liang, F., Brunelli, M., & Rezaei, J. (2019). Consistency Issues in the Best Worst Method: Measurements and Thresholds. *Omega*, 102175.
- Mohammadi, M., & Rezaei, J. (2019). Bayesian best-worst method: A probabilistic group decision making model. *Omega*, 102075.

And visit this website:

http://www.bestworstmethod.com

Here you can also find the ways (such as an Excel Solver) you can solve your BWM problems.

Or contact me: j.rezaei@tudelft.nl; info@bestworstmethod.com

